TWO-LOOP JET PHYSICS: STATUS AND PROSPECTS [*]

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I review the recent theoretical progress towards the computation of jet observables at two loops in QCD.

1 Introduction

Jet production observables are among the most sensitive probes of QCD at high energy colliders, where they are used for example to determine the strong coupling constant. At present, the interpretation of jet production data within perturbative QCD is restricted to next-to-leading order (NLO) calculations, with theoretical uncertainties considerably larger than current experimental errors. The extension of jet calculations to NNLO requires various ingredients, such as two-loop corrections to multi-leg amplitudes, multiple collinear limits of tree amplitudes, as well as a consistent method for the numerical computation of jet observables from NNLO parton level cross sections. In this talk, I review recent progress made on these subjects.

Jet observables of particular phenomenological relevance correspond to $2 \to 2$ scattering or $1 \to 3$ decay kinematics. They are:

- Three-jet observables in e^+e^-

- DIS (2+1)-jet production

- Hadron-hadron 2-jet and $(V + 1)$-jet production

The experimental accuracy on these processes at LEP, HERA and the Tevatron has already reached a level that makes theoretical predictions beyond NLO desirable.

2 Structure of NNLO jet physics

At leading order in perturbation theory, a jet is approximated by a single parton. Each parton in an event is required to be well separated in phase space from all other partons. Extending jet calculations to higher orders, this simple picture is no longer true.

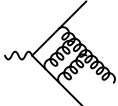[*]TALK PRESENTED AT 9TH INTERNATIONAL WORKSHOP ON "DEEP IN-ELASTIC SCATTERING AND QCD" (DIS 2001), APRIL 2001, BOLOGNA, ITALY
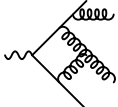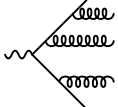
Subprocess	Partonic final state	Partons in jets
$\gamma^* \to 3$ partons, 2 loop e.g.	3 partons	(1) (1) (1)
$\gamma^* \to 4$ partons, 1 loop e.g.	4 partons (3 +1) partons	(2) (1) (1) (1) (1) (1)
$\gamma^* \to 5$ partons, tree e.g.	5 partons (4 +1) partons (3 +2) partons	(3) (1) (1) (2) (2) (1) (2) (1) (1) (1) (1) (1)

Figure 1. Contributions to $\gamma^* \to 3$ jets at NNLO; $(m+n)$ partons indicate m theoretically resolved and n theoretically unresolved (soft or collinear) partons.

At NLO, one has to include one-loop virtual corrections to the leading order subprocess as well as corrections from one-particle real emission. Both these contributions contain divergences, which have to be extracted, usually using dimensional regularization [1,2], before a jet algorithm is applied to the different final states in order to compute a particular jet observable. Since the jet algorithm acts differently on each partonic final state, it is not possible to combine contributions from real and virtual emission, as is done in multi-loop calculations of fully inclusive observables. It is for this reason that calculations of jet observables were limited to NLO accuracy up to now.

At the next-to-next-to-leading order (NNLO), one has to consider three different contributions: two-loop corrections to the leading order subprocess, one-loop corrections to the one-particle real emission, as well as two-particle real emission. Figure 1 shows the different contributions to three-jet production in e^+e^- annihilation. All three contributions individually contain divergences of infrared origin, which cancel only in the sum of the contributions. Again, each contribution has to be evaluated separately, since the (experimental) jet algorithm acts differently on final states with different numbers of partons.

3 Real corrections

Real corrections at NNLO consist of one-loop corrections to one-particle emission processes and of two-particle emission processes. Since the infrared divergences of these real corrections have to be extracted before application of the jet algorithm, it is necessary to introduce an infrared separation procedure, such as phase space slicing, subtraction, or a hybrid of these two methods.

At present, general algorithms exist for the computation of the one-loop corrections to one-particle emission processes [3] both in the framework of slicing and of subtraction methods. The computation of all two-particle emission contributions relevant to jet physics still is, however, an open issue. Some of these contributions have been computed within the hybrid subtraction method in the context of the calculation of the photon+1 jet rate in e^+e^- annihilation [4]. This calculation is moreover the only example so far of a numerical implementation of NNLO corrections to a jet observable.

4 Virtual corrections

The jet production observables listed in the introduction correspond to $2 \to 2$ scattering or $1 \to 3$ decay kinematics, i.e. the relevant scattering amplitudes are four-point functions with massless internal propagators and up to one external leg off shell. The large number of different integrals appearing in the corresponding two-loop Feynman amplitudes can be reduced to a small number of master integrals. The techniques used in these reductions are integration-by-parts identities [2,5] and Lorentz invariance [6]. A computer algorithm for the automatic reduction of all two-loop four-point integrals has been derived [6,7].

For two-loop four-point functions with massless internal propagators and all legs on shell, which are relevant for example in the NNLO calculation of two-jet production at hadron colliders, all master integrals have been calculated over the past two years. The calculations [8,9] were performed using the Mellin–Barnes method and the differential equation technique [10]. The resulting master integrals can be expressed in terms of Nielsen's generalized polylogarithms [11]. Very recently, these master integrals were already applied in the calculation of two-loop virtual corrections to Bhabha scattering [12], in the limit of vanishing electron mass, and to parton–parton scattering [13]. To obtain the full virtual corrections, one also needs to know the corresponding squared one-loop amplitudes [14].

We have used [15] the differential equation approach to compute all master integrals for two-loop four-point functions with one off-shell leg. Earlier par-

tial results on these functions were available in the literature [16,17], as well as a purely numerical approach to their computation [18]. We find full agreement with these earlier results. Our results [15] for these master integrals are in terms of two-dimensional harmonic polylogarithms (2dHPL), a generalization of the harmonic polylogarithms [19]. All 2dHPL appearing in the divergent parts of the master integrals can be expressed in terms of Nielsen's generalized polylogarithms of suitable non-simple arguments, while the 2dHPL appearing in the finite parts are one-dimensional integrals over generalized polylogarithms. An efficient numerical implementation of these functions is currently being worked out. Our results correspond to the kinematical situation of a $1 \to 3$ decay, their analytic continuation into the region of $2 \to 2$ scattering processes requires the analytic continuation of the 2dHPL, which is outlined in [15].

These four-point two-loop master integrals with one leg off shell are a crucial ingredient to the virtual NNLO corrections to three-jet production in e^+e^- annihilation, two-plus-one-jet production in ep scattering and vector-boson-plus-jet production at hadron colliders.

5 Summary and Outlook

Considerable progress towards the computation of jet observables has been made recently, concerning in particular the analytic calculation of the virtual two-loop corrections to four-point amplitudes: all two-loop amplitudes for parton–parton scattering are now known, and all master integrals relevant to the decay of a vector boson into three partons have been calculated.

However, these corrections form only part of a full NNLO calculation, which also has to include the one-loop corrections to processes with one soft or collinear real parton as well as tree-level processes with two soft or collinear partons. While a general procedure exists for the calculation of the former, considerable effort is still needed to derive the latter.

Only after summing all these contributions (and including terms from the renormalization of parton distributions for processes with partons in the initial state) do the divergent terms cancel one another. The remaining finite terms have to be combined into a numerical programme implementing the experimental definition of jet observables and event-shape variables.

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